

## SECONDARY INFALL: THEORY *VERSUS* SIMULATIONS

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### ABSTRACT

The applicability of the highly idealized secondary infall model to ‘realistic’ initial conditions is investigated. The collapse of proto-halos seeded by  $3\sigma$  density perturbations to an Einstein–de Sitter universe is studied here for a variety of scale-free power spectra with spectral indices ranging from  $n = 1$  to  $-2$ . Initial conditions are set by the constrained realization algorithm and the dynamical evolution is calculated both analytically and numerically. The analytic calculation is based on the simple secondary infall model where spherical symmetry is assumed. A full numerical simulation is performed by a Tree N-body code where no symmetry is assumed. A hybrid calculation has been performed by using a monopole term code, where no symmetry is imposed on the particles but the force is approximated by the monopole term only. The main purpose of using such code is to suppress off-center mergers. In all cases studied here the rotation curves calculated by the two numerical codes are in agreement over most of the mass of the halos, excluding the very inner region, and these are compared with the analytically calculated ones.

The main result obtained here, reinforces the findings of many N-body experiments,

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is that the collapse proceeds 'gently' and not *via* violent relaxation. There is a strong correlation of the final energy of individual particles with the initial one. In particular we find a preservation of the ranking of particles according to their binding energy. In cases where the analytic model predicts non-increasing rotation curves its predictions are confirmed by the simulations. Otherwise, sensitive dependence on initial conditions is found and the analytic model fails completely. In the cosmological context power spectra with  $n \geq -1$  yields (in the mean) non-increasing rotation curves, and in such cases the secondary infall model is expected to be a useful tool in calculating the final virialized structure of collapsing halos.

## I. INTRODUCTION

In a cosmological model in which structure forms via gravitational instability, the collapse of proto-structures onto local density maxima of the primordial perturbation field plays a major role in structure formation. Although the final (non-linear) density maxima do not necessarily emerge out from initial (linear) density maxima (*cf.*, Hoffman 1986, and 1989, Katz *et al.* 1993, Zaroubi and Hoffman 1993b and Bertschinger & Jain 1994) there is a good evidence to believe a strong connection between initial high maxima and final density maxima (van der Weygaert & Babul 1994).

The problem of collapse seeded by such maxima has been investigated from two different points of view. One is the statistical distribution of the objects formed this way, namely the distribution of galaxies and clusters. This is related to the so-called biasing problem which has been extensively studied (Kaiser, 1984, Davis *et al.* 1985 Bardeen *et al.* 1986 hereafter BBKS). The other aspect of the collapse seeded by local peaks is the internal structure of the objects thus formed and its dependence on the statistical properties of the primordial perturbation field. This problem had been first addressed by Gott & Gunn (1972) and was followed by the seminal paper of Gunn (1977, hereafter G77). The ideas and formalism suggested by G77 have led to considerable efforts to understand the

dynamical and statistical aspects of the problem, using both analytical calculations and numerical N-body simulations to understand this highly non-linear collapse process. It is the aim of the present paper to present N-body simulations which have been designed to check and make detailed comparison with the predictions of the analytical calculations.

Gunn & Gott (1972) studied the very idealized problem of the collapse onto a single  $\delta_{\text{Dirac}}$ -like density perturbation to an otherwise unperturbed flat Friedmann universe. This process has been described later on as the secondary infall, namely the collapse of less bound shells onto a perturbation that has already collapsed and virialized. By following the trajectories of spherical shells, and ignoring the process of shell crossing, the final virialized structure was calculated. The major breakthrough was done by G77 who realized that the spherical collapse onto the peak admits an adiabatic invariant. The case of a self-similar perturbation embedded in a flat Einstein–de Sitter universe allows an analytical calculation of the final virialized structure which takes into account shell crossing. The pioneering work of G77 was extended by Filmore and Goldreich (1984, hereafter FG) and Bertschinger (1985). These authors presented a new numerical approach to the problem of self-similar collapse in a flat universe. These self-similar solutions provide an exact description of the collapse all the way to the final virial equilibrium. FG further extended the analytical approach of G77 and presented an asymptotic analytical solution of the density structure, the nature of which is described below. A similar analytical calculation was presented also by Bertschinger and Watts (1988).

The problem of collapse and in particular the formation of dark halos, hereafter this term describes bound objects made of collisionless matter, was studied in great detail by Quinn, Salmon and Zurek (1986; QSZ), Zurek, Quinn and Salmon (1988), Frenk *et al.* (1988) and Crone, Evrard and Richstone (1994). These authors addressed the problem by means of N-body simulations, in which the full non-linear evolution from typical initial conditions corresponding to different models was followed. The main emphasis of these papers is on the dynamical resolution of forming objects and the study of their structure.

Thus, unlike the analytical approach where very idealized physical systems have been considered, the N-body simulations enable the study of the evolution of typical objects whose structure is not self-similar and does not obey a high symmetry. These studies focused on finding systematic trends in the forming structures, and in particular possible dependence on the initial conditions.

In the standard model of cosmology structure emerges out of a primordial perturbation field. This field is assumed to constitute a random Gaussian field, whose statistical properties are defined by its power spectrum. A first step to bridge the gap between the highly idealized analytical calculations and the numerical simulations was taken by Hoffman and Shaham (1985, hereafter HS), who calculated the ensemble mean density profile around high local density maxima. Assuming that structure around peaks of the primordial field is indeed given by the ensemble average, and that the dynamics is well approximated by the G77 dynamical model, HS calculated the virialized final structure expected in a variety of cosmological models. The analytical calculations of HS present a highly simplified picture of the extremely asymmetrical and very non-linear actual physical process. In spite of this high level of simplification, the predictions made by HS have been basically confirmed by N-body simulations (QSZ, Frenk *et al.* 1988). Hoffman (1988) made a detailed comparison of analytical predictions of the so-called secondary infall model (hereafter SIM) with the simulations of QSZ and Quinn and Zurek (1988) and found a good agreement in terms of the rotation curves of dark halos that are formed in the simulations. Thus, the simple model successfully reproduces the virialized structure of halos. However, a closer inspection of the formation process of these halos reveals a picture that stands in complete disagreement with the SIM. Halos do not form by the infall of spherical shells, but rather by the coalescence and mergers of substructures (QSZ). Nothing of the symmetric and ordered process envisaged by HS is actually confirmed by the numerical simulations, in which the collapse process of halos seems to be an extremely unordered and random process. In spite of these two very different formation pictures, the analytic

and numerical calculations yield a similar final structure. It is this seemingly paradoxical behavior that the present *paper* addresses, by designing and running N-body calculations so as to probe the basic ingredients of the collapse process of halos. The main aim of the *paper* is to find the key to the success of the SIM, and at the same time to determine the limitations of this simple model. A major goal here is to define under what conditions the model can be applied to realistic initial conditions and to serve as a practical model for bound structure formation in an expanding universe. An entirely different approach to the problem has been adopted by Quinn and Zurek (1988) and Warren *et al.* (1991), who suggested that the basic mechanism is related to the mergers of subclumps rather than centralized collapse. The role of mergers will be studied here by the N-body simulations. The structure of the *paper* is as follows. In §II the main features of the analytical model are presented and the basic question which are to be solved by the numerical simulations are posed. This is followed by the description of the numerical simulations, in §III, and the initial conditions, in §IV. The results of the N-body simulations are analyzed in §V. The *paper* concludes with a discussion given in §VI.

## II. Secondary Infall

The main essence of the SIM is that for a single spherical scale-free perturbation to an otherwise homogeneous flat universe, the final time averaged radius of a given Lagrangian shell scales with its initial radius. Thus, for a primordial power law density perturbations to an Einstein-de Sitter universe one expects a self-similar evolution. For the case of a Gaussian perturbation field the mean density profile around high local maxima of the density perturbation field is given by the autocorrelation function. In the case of a scale free field whose power spectrum is given by  $P(k) \propto k^n$ , where the relevant exponents in cosmology are in the range of  $-3 < n < 1$ , one finds (HS):

$$\delta(r_i) \propto r_i^{-(3+n)}, \quad (1)$$

(Here  $r_i$  is the initial radius defined at some arbitrary time, well within the linear regime.)

A fiducial density profile is defined as the structure one would obtain by freezing all shells at their turn-around radius. This yield (HS):

$$\rho_0(r) \propto r^{-\gamma_0}, \quad (2)$$

where

$$\gamma_0 = \frac{3(3+n)}{4+n}. \quad (3)$$

Assuming that the final (time averaged) radius  $r$  of the shell labeled by  $r_i$  scales with  $r_i$ , one finds that the final density run,  $\rho(r)$  scales with the fiducial one, namely  $\rho(r) \propto r^{-\gamma_0}$ .

In the case of a centralized perturbation, whose inner part is denser than its outer regions, one expects a secondary infall, namely the inner denser part collapses and relaxes first and the outer shells are collapsing on a longer time scale. As was pointed first by G77, the dynamics of the infalling less bound shells admits the product  $rM(r)$  as an adiabatic invariant, where  $M(r)$  is the mass that is momentarily interior to the radius  $r$ . Expressing the scaling of the final radius with the initial one by relating  $r_f$  to the turn around radius  $r_0$  we write  $r_f = Fr_0$ , where for a self similar infall  $F$  is a constant that depends on  $\gamma_0$ . Now, the one-to-one dependence of  $r_0$  on  $r_i$  is easily found from energy considerations. Given all that, the adiabatic invariance implies that the rotation curve of the halos is given by:

$$V_{rot}^2 = \frac{GM_f(r_f)}{r_f} = \frac{GM_0(r_0)}{F^2 r_0} \quad (4)$$

Here  $M_0(r_0)$  is the mass enclosed by the shell labeled  $r_i$  at the turn around epoch. To complete this analytical calculation one should find the collapse, or contraction, factor  $F$ .

For spherical systems, the mass interior to a given, *i.e.*, Lagrangian shell, is conserved up to its turn around. However, beyond this stage shell crossing occurs and this mass is not conserved any longer. The equation of motion of spherical shells admits a self-similar solution which transforms the partial differential equation into an ordinary one, which is solved numerically (FG, Bertschinger 1985). These numerical exact solutions can be obtained analytically in the asymptotic limit of a shell which has past its turn around phase

much earlier than some given time, defined as the ‘present’ time, and whose apocenter is much smaller than the current turn around radius,  $R_0$ . These asymptotic solutions have been solved subject to the approximations of spherical symmetry, scale invariance and the validity of adiabatic invariance (G77, FG, Bertschinger and Watts 1988). This problem has been recently addressed and an exact analytical solution for the collapse factor  $F$  in the asymptotic limit has been obtained (Zaroubi and Hoffman 1993, hereafter ZH). This solution is presented here and used in the comparison of analytical and numerical calculations. Consider a shell whose current apocenter (which is approximately equal to the time-averaged radius) is  $r$  and whose turn around radius is  $r_0$ , then the collapse factor is given by:

$$F = \left( 1 + (3 - \gamma_0) \int_1^U u^{2-\gamma_0} P\left(\frac{1}{u}\right) du \right)^{-1} \quad (5)$$

Here  $P(r/r')$  is the fraction of time a particle with apocenter  $r'$  spends inside  $r$ ,  $U = R_0/r_0$ ,  $R_0$  is the outer radius of the system or the present epoch turn-around radius and  $r_0$  is the turn-around radius of the shell whose current apocenter is  $r$ . The crucial step is the evaluation of  $P(u)$ . Various approximations were taken by G77, FG and Bertschinger (1985) and it has been solved exactly by ZH. Now, the integral in the denominator of Eq. 5 shows a very different behavior for  $\gamma_0$  smaller or larger than 2 (which corresponds to  $n = -1$ , in the limit of self similar systems). In the limit of  $U \gg 1$  the integral is dominated by its lower limit for  $\gamma_0 > 2$  and by its upper limit for  $\gamma_0 < 2$ . Thus for  $\gamma_0 > 2$  the dynamics of the shell labeled by  $r$  is affected mainly by nearby outer shells such that  $r' \gtrsim r$ , while in the other case of  $\gamma_0 < 2$  it is dominated by shells that have turned around only recently,  $r'_0 \lesssim R_0$  and  $r'_0$  is the turn around radius corresponding to  $r'$ . It follows that in the above limit one finds self-similarity for  $\gamma_0 > 2$ . In the other case of  $\gamma_0 \leq 2$  the final density run is a power law of 2. Note, that in the case of  $\gamma_0 > 2$  the effect of shell crossing is a local one, as only nearby shells of  $r' \gtrsim r$  contributes to the mass interior to  $r$ . Therefore, in such a case one might hope that the model can be applied over a finite

range of  $r$ , even if globally the structure does not follow a power law and self-similarity. For  $\gamma_0 \leq 2$  the main contribution to  $M(r)$  comes from the outer shells that have turned around only recently. This introduces a very sensitive dependence on boundary conditions. The appropriate upper limit for the integral of Eq. 5 is  $U \approx 3$  (ZH). For such a value the dependence of  $F$  on  $\gamma_0$  is fitted by:

$$F = 0.186 + 0.156\gamma_0 + 0.013\gamma_0^2 + 0.017\gamma_0^3 - 0.0045\gamma_0^4 + 0.0032\gamma_0^5 \quad (6)$$

The analytical model presented here relies on three basic assumptions, namely spherical symmetry, scale invariance and the validity of the adiabatic invariance. Yet, for actual halos emerging out of Gaussian random fields it is clear that at least the first two assumptions do not apply. The structure around typical peaks is neither spherical nor is it characterized by a power law (BBKS) and it is certainly finite. The question thus arises is to what extent can the simple model describe the complex dynamical evolution. Now, from previous numerical simulations we know that at least in a statistical sense the model does correctly yield the final structure of bound halos (QSZ, Frenk *et al.* 1988; for detailed comparison see Hoffman, 1988). Thus, applying the model to ensemble averaged density profile reproduces the ensemble average taken over all the objects in the simulations, the dynamics of each one of which is properly calculated by the N-body codes.

### III. Numerical Simulations

We model here the collapse of bound structures seeded by local density maxima of the primordial perturbation field in a flat Friedmann universe. The perturbation field is assumed to be a Gaussian random field which is defined by its power spectrum. For a scale free process the power spectrum is given by  $P(k) \propto k^n$ . Given a particular realization of the perturbation field, constrained to be centered on a local density maximum, the subsequent dynamical evolution is calculated by three different methods.

First, the final virial configuration is calculated by using the SIM. This is done by taking the spherically averaged  $\delta$ -field (centered on the peak), and applying Eq. 4 to it.

Thus, the analytical model that has been derived for scale-free initial conditions (ZH), is applied here to a general initial density profile which is not self-similar. The collapse factor  $F$  is calculated here for each given spherical shell of radius  $r$  by using the local logarithmic derivative of the initial density profile. The self-similar solutions are applied here locally, as if the shell crossing process depends on the local properties of the density field. In the case where the actual density profile yields a fiducial density which follows an  $r^{-\gamma_0}$  power law with  $\gamma_0 > 2$ , the actual  $\gamma_0$  thus obtained is used to calculate the collapse factor  $F$ . However, for  $\gamma_0 < 2$  we take it to be  $\gamma_0 = 2$ . The dependence of  $F$  on  $\gamma_0$  is calculated by ZH. This relation has one free parameter, namely the ratio of the largest turn-around radius (at a given time) to the turn-around radius of the shell under consideration. Based on the N-body simulations here we estimate this ratio to be  $U = 3$  in average.

Second, the final virialized halo is calculated by evolving the initial system in full N-Body simulations. These N-body simulations are carried out by using the Treecode algorithm (Barnes and Hut, 1986, and Hernquist 1987, 1988, 1990). Third, we introduce a new numerical model to trace the evolution of high density peaks under the assumption of spherically symmetric force and realistic initial conditions, rather than the spherically symmetric force and initial conditions as assumed in the SIM. This code is used to trace the evolution of a single peak in an otherwise Einstein-de Sitter universe. Such a code stands in between the full N-body calculation and the simplified analytical model. It turns out that the rotation curves of the virialized systems calculated using this method or with the Treecode, agrees well over most of the halos, excluding the very inner regions (see §V). Therefore this method is used here as a reference for testing the SIM predictions for most of the simulations.

The location of the center of the peak is very crucial for this kind of approach and it should be carefully defined; here we chose it to coincide with the point of maximal density. The time step used in this code is taken to be a fixed fraction from the dynamical time ( $\tau_{dyn}$ ) of the system. Where  $dt = 10^{-4}\tau_{dyn}$  gives a very good energy conservation

$(\frac{\Delta E}{E} \leq 1\%)$  over a complete simulation. A softening parameter is used to avoid an infinite force at the origin. The simulated region has been chosen to be a sphere of radius  $R = 1$ , which along with fixing the gravitational constant to unity ( $G = 1$ ) define the physical units of the simulation. The softening parameter of most of the simulations is 0.1.

The two basic limitations of the present simulations are the nature of boundary conditions and the rather small number of particles. Here vacuum boundary conditions are assumed and therefore tidal interactions are ignored. As for the number of particles, in most runs  $N = 2000$  particles are used. For one of the runs this has been increased to  $N = 4000$  and  $8000$ , in such a way that the increase in the number of particles was used to increase the extent of the simulated objects and not the code resolution. Now, these limitations certainly affect the outcome of the simulations. However, given that the simple model has already been successfully confirmed by large scale simulations (*e.g.*, QSZ), in which boundary conditions are properly handled, and given that our simulations are aimed at studying in a controlled way the dynamics of individual objects, we think that the present calculations are adequate for addressing the problems stated here.

#### IV. Initial Conditions

The formalism of constrained realizations of Gaussian random fields (Hoffman & Ribak 1991) has been used to set the initial conditions. Here we are interested in making a particular realization of a Gaussian perturbation field, which is constrained to have a  $3\sigma$  density peak located at the center of the computational box. A peak is specified by 10 constraints, namely the peak amplitude, the three first partial derivatives which are set to zero, the three eigenvalues of the second-order derivatives which form a  $3 \times 3$  symmetric matrix, and the three angles which define the direction of the eigenvectors. The three eigenvalues which define the shape of the peak are taken to have their mean values (BBKS), and the eigenvectors are taken along the Cartesian axis of the computational box. The realizations are designed to have such peaks at the center, but they have also other

unconstrained local maxima and minima.

The physical systems simulated here are self-similar in the mean. The background universe is Einstein-de Sitter and the power spectra considered here are scale free,  $P(k) \propto k^n$   $\{n = -2, -1, 0, 1\}$ . Two sets of random numbers, which specify the phases and amplitudes and hence the given realization, are constructed here and are used for the different power spectra considered. The two realization are labeled as *a* and *b*. The number of particles used for each simulation is typically  $\approx 2000$  except in two cases where we used 4000 and 8000 particles with the hybrid code to test the effect of larger systems.

A summary of the numerical experiments calculated here is given in Table I. The simulations vary with respect to power spectrum (*n*), family *a* or *b* of realizations, number of particles, and the kind of code used. All models were calculated with the hybrid (symmetrical) code, some of which were calculated also by the full Tree-N-body code.

[ **Table 1** ]

## V. RESULTS

Here we are interested in the gross density structure of the forming halos and this is described by the rotation curves, thus ignoring much of the fine structure. The resulting rotation curves calculated in the three different ways are given in Figs. 1(a-c) of three models of family *b* with power spectra of  $n = -2, -1, 0$ . Other models for which the rotation curves are calculated by the (analytic) secondary infall and the hybrid numerical algorithms are given in Fig. 2(a-c). A close inspection of the calculated rotation curves shows, first, the good agreement between the full N-body and hybrid codes over most of the mass of the halos. There is a discrepancy at the very inner region of the systems. The comparison of the analytically expected and numerically calculated curves is more difficult to interpret. In cases where the local effective  $\gamma_0$  is smaller than 2, the rotation curve is predicted to increase with radius, corresponding to a diverging mass profile, and the structure is determined by the latest infalling particles. Not surprisingly the numerical

simulations cannot reproduce such rising rotation curves; this has also been confirmed by higher resolution N-body experiments of QSZ and Crone *et al.*(1994). However, for models with spectral index larger than  $-1$  the effective  $\gamma_0$  increases and a better agreement is expected. Indeed, in the case of the two realizations  $(a, b)$  of  $n = 0$  and in particular  $n = 1$  the predicted rotation curves are close to the simulated one. Yet, in all cases the numerically calculated curves fall below the analytical ones, indicating a larger collapse factor ( $F$ ) than the calculated factor.

The N-body simulations of halos with predicted rising rotation curves poses severe numerical problems because of its extreme sensitive dependence on the boundary conditions. Previous simulations of a representative computational box that includes many halos and an  $n \leq -2$  power spectrum yielded non increasing rotation curves (*e.g.*, QSZ). To study this problem the  $n = -2$  model ( $b$ ) has been further simulated with an increasing number of particles,  $N = 2000, 4000$  and  $8000$ , using the hybrid code. The added particles are used to simulate a larger initial volume and not to increase the dynamical resolution. The outcome is presented in Fig. 3, where the calculated rotation curves are presented and compared with the expected curve for the  $N = 8000$  simulation. The change of the simulated curves with  $N$  is expected. In the  $N = 8000$  case we find a flat rotation curve over about the inner half mass that is followed by a decreasing curve. Note that the predicted curve is now dominated by the outer shells and is decreasing. This manifests the sensitive dependence on the boundary conditions and shows that the SIM fails in the  $\gamma_0 \leq 2$  case. The model is validated, however, in the  $\gamma_0 > 2$  regime of declining curves.

A possible explanation of this surprising agreement between the analytic model and the simulations lies in the ordered behavior of the collapsing systems in energy space as opposed to the random behavior in real (configuration) space (Hoffman, 1988). As was noticed by Quinn and Zurek (1988) the seemingly very complicated collapse process looks very ordered and 'gentle' when viewed in energy space. This leads us to the key question to be addressed here, namely to what extent a collapsing system 'remembers' its initial

conditions. In particular we focus here on the energy (per unit mass) behavior of individual particles, which changes in time as  $(d/dt)(\phi + v^2/2) = (\partial/\partial t)\phi$ . Therefore in collapsing systems the energy of particles changes in time, however it is not clear *a priori* whether this change is coherent or leads to a substantial phase mixing. This is studied here by drawing scatter plots of the final *vs.* the initial energies of individual particles in all the hybrid-code simulations (Figs. 4 (a-f)). The full Tree-N-body simulations yield similar scatter plots. A strong correlation exists in all cases, indicating a coherent and ordered evolution in energy space. It is clear that such systems do not go through an efficient phase mixing, and thus they retain the memory of their initial conditions. Note that there is a clear connection between this correlation and the spectral index  $n$ , the tightest correlation corresponds to the  $n = 1$  case and it decreases with  $n$ . This trend coincides with the dependence of the agreement between the SIM and the numerical simulations on the spectral index.

To further study the energy evolution we rank the particles according to their binding energies, from the least to the most bound particle. The possible change of this ranking throughout the collapse and virialization process is studied by plotting histograms of the frequency of the relative change of this ranking of all particles of a given hybrid-code simulation Figs. 5(a-g). One finds that in all cases some 70% of particles do not change their rank by more than 10%, and a very similar result is found in the full Tree-N-body simulations. The approximate conservation of the ranking throughout the dynamical evolution proves that the cosmological collapse proceeds rather 'gently' and involves no violent relaxation.

Next, the trajectories of individual particles are considered. In Figs. 6(a-b) the trajectories in the  $(r - t)$  plane of 20 randomly selected particles from the (-1;b) model are presented. Note, that this is the most successful model in terms of reproducing the numerically calculated rotation curve. Yet, these trajectories are in complete disagreement with the ones calculated by FG and envisaged by the simple analytic model. Very few trajectories do show the gradual shrinking of the turn-around radius. In the light of

this disagreement, the success of SIM in predicting the the correct rotation curves seems to be very surprising. However one can argue that the rotation curves depend mainly on the energy distribution, which is much more smoother and more rounded than the underlying density distribution, that can account for the agreement between the model and the simulations (see §VI for more detailed discussion).

## VI. DISCUSSION

The SIM has been rigorously formulated for self similar systems, yet in the canonical cosmological model structure arises from a random perturbation field and proto-structure are necessarily finite and not self-similar. The question that is then naturally asked is to what extent this highly symmetric model is applicable to ‘realistic’ initial conditions. Previous numerical work (*e.g.*, QSZ) has already proved the validity of the model when applied to N-body simulations and that in general the simple model correctly predicts the structure of bound objects, at least in the statistical sense. (This holds in the  $n \geq -1$  case.) Yet, closer inspection of these simulations shows that the actual collapse process seems to be in complete disagreement with the one predicted by the simple model. It is this ambivalent behavior that has been studied here. Using the algorithm of constrained realizations special care has been given to the setting of the initial conditions of the simulated objects. Given the initial configurations, these were evolved dynamically to virial equilibrium in three different ways, namely by ‘exact’ N-body simulations, the analytical model and the hybrid N-body ‘monopole term’ code. Thus, a given initial configuration has been evolved by three different methods and the final results are compared. Our basic conclusion is that within the limitation of the numerical simulations the gross agreement between the model and the large scale N-body simulation is confirmed at the level of individual objects, in cases where such agreement is expected.

Our main motivation here is to perform controlled numerical ‘experiments’ of the collapse onto local density maxima, with an emphasis on the controlled aspect. This is

achieved here in two ways. One is the use of constrained realizations for setting the initial conditions. The other is the use of the hybrid monopole term code that stands between the highly symmetric and analytic SIM and the full N-body simulation. The comparison of the outcome of this code with that of the standard Tree N-body code allows us to separate the dynamical effects from that of (the lack of) spatial symmetry. We find here that within the technical limitation of our simulations, namely the lack of tidal interactions, the two modes of simulations are in a good agreement. This proves that mergers mechanism does not stand at the basis of the success of the SIM.

The basic physical understanding that emerges out of the present collapse ‘experiments’ is that of two different dynamical regimes depending on the logarithmic derivative of the (spherically averaged) fractional overdensity as a function of radius, *i.e.*, the effective power law index  $n$  or equivalently the effective  $\gamma_0$  of the fiducial turnaround density profile. In the cosmological context these regimes correspond to a primordial perturbation field whose power spectrum is dominated by the high wave number modes  $n \geq -1$  or low wave number modes  $n \leq -1$ . In the regime of highly correlated perturbation field,  $n \leq -1$  (*i.e.*, in the mean  $\gamma_0 < 2$ ), there is a strong dependence on the boundary conditions, the dynamics strongly depends on the last collapsing shells. In such cases the energy of particles changes violently in time and one expects to reach at the end a state of statistical equilibrium, much in the spirit of the Violent Relaxation proposed by Lynden-Bell (1967). It is important to note that the two very different approaches, namely the violent relaxation and the SIM, predict the same final structure of an asymptotic  $r^{-2}$  density profile in the limit of infinite spherical system. In the other regime of  $n \geq -1$  the primordial structures are sharply peaked and the dynamics of already collapsed shells is hardly affected by the ongoing collapse of more distant and less bound shells. In such a case order is being preserved in energy space, the initial conditions are well ‘remembered’ by the system and the final structure reflects the initial conditions in a manner described here.

The regular and ordered evolution in energy space suggests a possible explanation as

to why a model based on spherical symmetry predicts correctly the final mass distribution of collapsing halos. There is a high correlation between the initial and final energies of individual particles, and in particular the ranking in energy space is roughly preserved. Now, the virial structure depends on, and is basically determined by, the final energy distribution, and consequently it depends on the initial energy distribution. In the linear theory of gravitational instability the total energy is determined by the gravitational potential, which is always much smoother and more rounded than the underlying mass distribution. Thus, the simple analytic model which seems at first glance to (incorrectly) describe the mass distribution, provides quite a good description of the initial energy distribution of particles, which in turn determines the final structure, in the case of  $\gamma_0 \leq 2$ .

The results presented here, and in particular the close agreement with the prediction of a spherical top-hat model, seem to be in conflict with earlier calculations on the role of shear in the gravitational collapse. These show that in the quasi-linear regime of gravitational instability in an expanding universe the shear (in the velocity field) accelerates the collapse and acts as a source of gravity (Hoffman 1986 and 1989, Zaroubi and Hoffman 1993b, Bertschinger and Jain 1994, Eisenstein and Loeb 1995). The shear, which is primarily induced by the tidal field, represents a deviation from spherical symmetry and thus its effect seems to be inconsistent with the present results. Based on this Bertschinger (1994) postulated that local density maxima are not the sites where collapse occurs first. This, and the numerical simulations of Katz *et al.* (1993), shed doubt on the (linear) peaks – halos association. Yet, the more recent simulations of van de Weygaert and Babul (1994), which were designed to study the effect of shear, have reaffirmed that high enough ( $\gtrsim 2\sigma$ ) peaks evolve to form halos. They found, however, that shear affects the outer envelopes of halos. The simulations presented here were not designed to study these effects and the limited dynamical range does not allow proper modeling of the external shear, however the internal shear which arise from the aspherical local matter distribution is well presented. This affects the collapse significantly in the quasi linear regime (Zaroubi and Hoffman,

1993b), yet the final virial structure is consistent with the top-hat model prediction. Note also that in N-body simulations where the computational box contains a typical patch of the universe with many proto-objects, both internal and external shears are properly sampled. Yet, such simulations basically confirm the predictions of the simple, top-hat spherical model (QSZ, Frenk *et al.* 1988, Crone, Evrard and Richstone, 1994). A tentative conclusion is that shear affects the dynamical evolution in the quasi-linear regime only, but the virial structure is hardly affected by it. However, this should be further tested in controlled N-body simulations.

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**TABLE 1**

The Parameters Of The Simulations				
Label	Power Spectrum	Family	Particles' #	Simulations
0;a	n= 0	a	2000	Symm.*
1;a	n= 1	a	2000	Symm.
-2;b	n=-2	b	2000	Symm. + Nbody†
-1;b	n=-1	b	2000	Symm. + Nbody
0;b	n= 0	b	2000	Symm.
1;b	n= 1	b	2000	Symm. + Nbody
-1;b,4	n=-1	b	4000	Symm.
-1;b,8	n=-1	b	8000	Symm.

† Nbody, indicates full N-body simulation

\* Symm., indicates hybrid code simulation

## Figure Captions

**Figure 1.** : The final rotation curves calculated in the three methods, the theoretical model (dashed lines), the hybrid monopole-term code (long dashed lines), and the full N-body code (solid lines). The curves are drawn for the models labeled (see table I) (a) -2;b, (b) -1;b, and (c) 1;b

**Figure 2.** : The final rotation curves calculated theoretically (dashed lines) and numerically by the hybrid code (long dashed lines). The curves are drawn for the following models (a) 0;a, (b) 0;b, (c) 1;a.

**Figure 3.** : Rotation curves of the  $n = -2$  model (labeled -2;b) calculated with the hybrid code for  $N=2000$  particles (solid line),  $N=4000$  (dotted line), and  $N=8000$  (small dashed line). This is compared with the theoretically expected rotation curve for  $N=8000$  (long-dashed line). Note that the length scale here differs from that of Fig. 1a.

**Figure 4.** : A scatter plot of the final energies compared with the initial ones of the bounded particles as calculated with the hybrid code. The different graphs corresponds to the following models (a) 0;a, (b) 1;a, (c) -2;b, (d) -1;b, (e) 0;b and (f) 1;b. The correlation coefficient is denoted on each graph.

**Figure 5.** : Histograms of the change in the ranking (by binding energy) of the particles in the energy space, as calculated by the hybrid code. The histograms show the percentage of particles which changed their ‘energy’ ranking within a certain percentage. The models here are the same as in Fig. 4.

**Figure 6.** : The radial trajectories of randomly selected particles in the Tree-N-body simulation for the model 1;b. All radii are normalized by the maximal turn-around radius of each model.